

Introduction to Mathematical Models in Economics

M.O.L. Bacharach
Christ Church, Oxford

M.A.H. Dempster
Dalhousie University
and Balliol College, Oxford

J.L. Enos
Magdalen College, Oxford

Editors

1. History of the Oxford Mathematical Economics Seminar

In academic circles, novel ideas are at times celebrated; new modes of teaching seldom, if at all. The present volume is an exception, commemorating as it does the twenty-fifth anniversary of the establishment of the Mathematical Economics Seminar at Oxford University. The advent of the Seminar, as we shall henceforth abbreviate it, was the summer of 1963, when Terence (W.M.) Gorman suggested to two of the graduates at Nuffield College that in the following term they should convene a class addressed to recent developments in economic theory. The purposes of such a class would be to extend the coverage of economic theory beyond that provided in the regular lectures for the B. Phil., as the initial graduate degree in Economics was then designated, and to generate ideas for subsequent D. Phil. theses.

The two graduate students prevailed upon were John Flemming and Richard Portes. Together they booked a seminar room in Nuffield College for the following term and persuaded several of their contemporaries to prepare papers for delivery. In Michaelmas term the class met, one session being held in each of the eight weeks of the conventional full term. Such was its appeal that the organizers were induced to continue the Seminar in the subsequent terms and, gradually, to invite the attendance of graduate students from other colleges of the University. Martin Feldstein, then a Fellow of Nuffield College, and John Black of Merton were involved with the first two years.

The whole venture illustrates one of the idiosyncrasies of Oxford: that almost anyone associated with the University can offer a set of lectures or a seminar. The University itself, and its constituent Colleges, offer accommodation in the form of lecture rooms; and the students, dons, visitors to the University and academically-inclined local residents provide the audience. The menu of attractions each term (the autumn term, called Michaelmas; the winter term, Hilary; and the spring term, Trinity) is printed in the University's "Lecture List"; the Mathematical Economics Seminar first appeared in this compendium in Trinity Term 1965.

The convenors of the Seminar in Trinity Term 1965 were John Flemming and John Black. Richard Portes, then economics fellow at Balliol College, rejoined the Seminar in Michaelmas Term, following which term John Flemming dropped out. Over the next three years there was a complete change of personnel, John Black leaving after Michaelmas Term 1966 and Richard Portes after Trinity Term 1968, to become professors at Exeter University and Princeton University respectively. Joining the Seminar, in chronological order, were Michael Dempster (from Carnegie-Mellon, in Hilary Term 1966), John Enos (from M.I.T., in Hilary Term 1967) and Michael Bacharach (from Cambridge University, in Hilary Term 1968). The convenors of the Seminar have remained the same for the intervening twenty years.

From time to time the Seminar has benefited from the presence of visitors, both from within the University and from foreign universities. Terence Gorman has joined in occasionally, most usefully when the Seminar lacked continuity or enthusiasm; others notably have been Brian Barry (Trinity Term 1968), James Mirrlees (a memorable year, Michaelmas Term 1968, and Hilary and Trinity Terms 1969), Avinash Dixit, for an extended and valuable period while economics tutor at Balliol College (calendar years 1970 through 1973), Joe Stiglitz (another memorable year, 1978-9), Colin McDiarmid (Trinity Term 1986), and Hyun-Song Shin (Trinity Term 1988). Visitors working in Oxford for a term or more who volunteered to help in organizing and conducting the Seminar were Dale Jorgenson (Hilary Term 1968), Robert Solow (the academic year 1968-9) and Eric Streissler (on three occasions, Hilary Term 1968, Trinity Term 1973 and Hilary Term 1979).

In the mid 1960s, around the time of the change of personnel, the format of the Seminar altered somewhat from the original format it shared with other Oxford graduate seminars in humanities and the social sciences. Instead of the graduate students reading formal papers, they spoke from notes which contained the main arguments and

mathematical developments and which were reproduced and handed out to each person in the audience. The Seminar then proceeded informally, with interruptions for clarification or for the more detailed exposition of content or technique. By the end of the 1960s, the Seminar had thus assumed a stable pattern—investigations of advanced topics in Economic Theory and Methods, beyond the requirements of the current B. Phil. (now M. Phil.) syllabus, addressed chiefly by graduate students, in the presence of others almost equally familiar with the material, in an informal setting where all could contribute.

What has defined an advanced topic in economic theory has generally been a combination of an interest on the part of the members of the Seminar, and several articles or a book utilizing mathematical methods and recently published. Rarely, the convenors of the Seminar have tried to initiate a topic, as in Trinity Term 1968 when the Seminar was devoted to economic theories of politics and in — — — ? — — — — to economic theories of animal behaviour and genetics. More often, and more successfully, the topics have been drawn from conventional areas of economic research or mathematical analysis.

Of the former, the most popular have been *models of economic growth* (Michaelmas Term 1967, Trinity Terms 1970, 1971, 1976 and 1982, and Michaelmas Term 1983), *uncertainty* (Michaelmas Term 1968, Hilary Term 1971, Trinity Term 1977, Hilary Term 1980 and Trinity Term 1988), *public economics* (Michaelmas Term 1964, Hilary Term 1965, Hilary Term 1969 and Trinity Term 1975), *macroeconomic theory and policy*, including *expectations* (Hilary Term 1977, Trinity Term 1978, Michaelmas Term 1979, Trinity and Michaelmas Terms 1980), and *capital theory* (Hilary Term 1968, Trinity Term 1973, and Hilary and Michaelmas Terms 1976). Three terms each have been devoted to *decision theory* (Michaelmas Terms 1976 and 1977 and Hilary Term 1978), *planning and decentralization* (Trinity Terms 1969 and 1972, and Michaelmas Term 1982), and *comparative economic systems* (Trinity Term 1968 and Michaelmas Terms 1975 and 1978). Theories of *income distribution* have occupied the Seminar twice (Trinity Terms 1974 and 1979), and *money* (Hilary Term 1973), *social choice theory* (Hilary Term 1975) and *optimal depletion of exhaustible resources* (Hilary Term 1979) once each.

Of the latter area, mathematical techniques, a total of eight terms have been devoted to familiarizing graduate students with those techniques useful in economics. In chronological order, these have been *linear algebra* and *mathematical programming* (Hilary Terms 1967 and 1970), *differential equations* (Hilary Term 1972), the *calculus of variations* and *optimal control* (Michaelmas Term 1974), *linear algebra*

and *mathematical programming* again (Trinity Term 1981), *game theory* (Hilary and Trinity Terms 1985) and, finally, *algorithm analysis* (Trinity Term 1986). In addition, a week or two has often been set aside for the elucidation of a mathematical technique within a term otherwise committed to an economic topic, e.g. two sessions on *stochastic processes* in a term in which recent developments in *macroeconomic theory* were addressed.

Given the frequency with which the Seminar has met—usually three terms per year, over the last twenty-five years—one might well ask what, if any, effect it has had on the development of the discipline. So far as the presentation of material is concerned, it has all been interesting, at least to those who have been convening the Seminar. Much of the material has been significant too, according to conventional standards, for it has been absorbed into the syllabus of the graduate programme in economics. The need to increase graduate students' facility in understanding mathematical language has also been recognized within the syllabus, resulting in the provision of a compulsory "crash course" in mathematics.

To argue that the Seminar has advanced the discipline of economics would be fanciful. It has been only one of several vehicles for the transport of new ideas at Oxford: the weekly Economic (and Econometric) Theory Seminar at Nuffield College to which economists passing through Oxford present the results of their own research is another, and the scheduled lectures on special subjects given regularly by members of the Economics Sub-Faculty are still others. The Seminar has been just one choice in a changing menu.

To the extent that it has fulfilled a unique role, the Seminar's contribution has been that of providing a university-wide proving ground for young economists. Drawing an audience from all of Oxford's thirty-seven colleges, soliciting speakers from the same multitude, the Seminar has offered every graduate student in economics the opportunity to investigate a topic, to extend or alter its coverage, to try out new ideas. For some, the mysteries first encountered in the Seminar have led to a career devoted to their penetration.

2. Introduction to the Volume

Many of the papers in this volume were written over ten years ago; a significant number were written even earlier. Indeed some were previously published up to three decades ago! Nevertheless, virtually all their authors are well known and include in their number three Nobel Laureates, two Chairmen of the Council of Economic Advisors of the US President, two Chief Economists of the World Bank, a Chief Economist of the Bank of England and a Senior Advisor to the Bank of Canada. It should therefore not be a surprise that most of the questions treated remain relevant to economics today. The twenty three contributions to the volume are grouped under three topics: theory (10 chapters), theory for policy (10 chapters) and policy (3 chapters).

The first five theoretical papers treat questions in the economics of information. Chapter 1 by Bacharach is a contribution which axiomatizes, in two different ways, the assumption in many economic models that rational agents' environmental knowledge is represented by an information partition. One approach to this question stems from formal systems termed 'epistemic logics', while the other formalizes statistical decision-theoretic notions of 'experiments' going back to Blackwell. Chapter 2 by Gorman is a contribution to the economic theory of organizations treated by Arrow in his Fels lectures in 1974 and subsequently developed from different perspectives by Williamson and North. The chapter gives an elegant mathematical treatment of which aggregate variables should be monitored by central management so that the operating units of a firm do not reduce the 'gains in the collection, dissemination and processing of information presumed to result from hierarchical organization of the firm'. Hammond in Chapter 3 turns to the study of the power of small coalitions to bring about Pareto efficient allocations through an appropriate concept of the core of competitive Walrasian markets by demonstrating appropriate limit theorems for large finite economics as the number of finite coalitions of fixed size tend to infinity. Chapter 4 by Stiglitz and Weiss continue this theme by examining the effects of information asymmetry on competitive equilibria which began with the seminal work of Spence. They contrast the properties of equilibria in which the informed agents signal the uninformed with those in which the uninformed agents screen the desirable agents from the less desirable with some surprising and often counterintuitive results depending on particular features of the markets studied. Farrell studies in Chapter 5 the perfect Bayesian equilibria of dynamic economic games of incomplete information in which an informed player may reveal information to an uninformed player through costless signals or cheap talk. The effectiveness of these signals is limited by their credibility and by the unlimited interpretations possible for those which would be a surprise in equilibrium – termed neologisms – and the paper analyses equilibria in which no credible neologisms are available. The next chapter (6) by Bray provides an account in accessible language of the ways in which uncertainty affects prices in financial markets. In a model in which agents make production decisions and

subsequently trade futures contracts on the resulting product, it is demonstrated that even with asymmetric information if all agents possess rational expectations about the underlying price process the futures market becomes informationally efficient and equilibrium is attained.

The remaining four theoretical chapters treat a variety of topics concerned with macroeconomic market imperfections. Chapter 7, by the Ulph brothers, studies a simple model of the labour market in which workers and firms have asymmetric information about macroeconomic shocks, but firms can control employment fluctuations by inventory control, factor substitution and capital choice. They show that in such a world that *ex ante* and *ex post* wage bargaining has real effects and leads to labour market imperfections through both the different bargaining powers of the firm relative to these short and long term ‘smoothing’ decisions and to its practical inability to sign long term labour contracts. Kay and Keen in Chapter 8 turn to the measurement of market imperfections induced by tax systems. They develop the implications of Debreu’s notion of inefficiency – as the fraction by which efficient inputs could be reduced relative to fixed current outputs – with regard to optimal individual lump sum taxes and show that tax distortions can be decomposed into individual and social components. In Chapter 9, Davis and Dempster consider the implications of a comparative static model of a developed economy in which as well as the usual macro economic relationships, there is a stock market financing investments, a labour market with wage bargaining and an endogenous government whose expenditures respond to the price level, GNP and unemployment. They show that such a model is – in various parameter regimes – capable of reproducing classical, Keynesian and monetarist outcomes and call for further study of the response of the price level to government deficit or surplus and of real activity to price level, interest rate and money stock in a dynamic setting. The stochastic dynamics of imperfectly competitive market structures arising from strategic innovation races are treated by Beath et al in Chapter 10 with a view to understanding when competitive technical innovation between initially asymmetric firms favours the dominant incumbent or its rivals. They identify both the distinction between product and process innovation and the combined effects of information dissemination and learning-by-doing as crucial to whether smaller rivals can leapfrog the incumbent in a process of Schumpeterian ‘creative destruction’ or are always doomed to ‘play catch-up’.

The second part of this volume also contains theoretical contributions, but they treat theory aimed at illuminating specific questions of economic policy and a few of these continue the themes of earlier chapters. The first five chapters of this part consider questions of national economic policy in a developed nation; the remaining chapters concern international competitive policy and policy in developing nations. Rankin, following Hart, continues the investigation of Chapter 8 into the determinants of an imperfectly competitive economy’s reaction to monetary and fiscal policy. He notes Wren-Lewis’ proposition that in such an economy levels of output and employment will depend upon these policies and studies

conditions under which standard Keynesian propositions apply. In Chapter 15, Roberts studies the stability of the long run full employment equilibrium which the model of the previous chapter precludes. He shows how wage and price changes in disequilibrium imply the instability of the full employment equilibrium due to a distribution of wages whose evolution over time is a new determinant of the business cycle. Chapter 13, by Solow, introduces a macro-economic disequilibrium model to explain the short term behaviour of the Phillips curve relating wages and prices in light of prior empirical work indicating the pass through of 'expected' price inflation to actual inflation. He demonstrates a number of possibilities for short-run equilibria including a demand limited stagflationary situation in which 'the price level (is) actually rising despite the existence of what any reasonable man would call excess capacity and unemployment'. Feldstein studies in Chapter 14 the long run effects of government deficits in a fully employed and growing economy using a three asset monetary growth model in which the government deficit is financed by issuing both money and interest-bearing debt. The chapter's conclusions are of considerable importance regarding recent economic history in their support of the reduction of taxes to stimulate investment so that 'with appropriate fiscal incentives and accommodating monetary policies, an increase in saving can be absorbed in greater capital intensity without any change in the rate of inflation' (*italics added*). In Chapter 15, Hausman and MacKie-Mason consider policy regarding incentives for technical innovation in an advanced economy. They show that in certain circumstances price discrimination by patent holders can be Pareto improving through allowing them to open new markets or achieve economics of scale or learning, while in others it may at least be more efficient than alternatives such as long patent lives.

The remaining five chapters of Part 2 of the volume treat questions of international competition and economic development. The first of these, Chapter 16 by Spence, studies industrial organization, evolution and competitive strategy with a view to evaluating policy attempting to enhance the competitive position of domestic versus foreign forms in a multinational industry. Three popular policies for strategic use are analysed: blocked access to domestic markets, subsidies and research and development (R&D) support. The last of these is also the topic of Chapter 17 by Dixit, who notes that domestic R&D cost reducing policies in a multinational industry can wipe out foreign R&D with negative domestic effects. Instead, strategic policies that restrict foreign R&D prospects may be more effective. Dreze and Stern address in Chapter 18 the question of assessment of public projects and policy reforms when market prices lead to revenues and costs which give distorted measures of social gains and losses. They extend the familiar notion of shadow prices for public sector to provide a coherent way of addressing privatisation and price reform issues in second-best – i.e. not fully competitive – economics. In Chapter 19, Dempster and Enos treat a dynamic model of a Lewis type underdeveloped economy which has a Leontief production technology. Using optimal control theory they obtain explicitly an optimal development policy for the shortest time to full employment and

apply the results to data for Thailand in the 1970s. The final chapter in this section, Chapter 20 by Portes and Santorum, is mainly an econometric study over three decades of post-revolutionary China of the relationships between macroeconomic variables such as money supply – treated endogenously – and the consumption goods market in. They conclude that money supply and income are co-determined, that narrow definitions of money are best for model building and that the Chinese economy has had regimes of both excess demand and excess supply over the sample period.

The final three chapters of the volume, constituting Part 3, are also econometric studies but support the analysis of specific policy issues. In the first of these, Chapter 21, Anand and Kanbur critically examine the methodology often used by the World Bank for international poverty projections since the early 1980s. They find that this methodology is not robust to sensible improvements and caution policy-makers and analysts to treat global poverty forecasts with caution. In Chapter 22, Fleming and Barr study the weekly dynamics of the UK money market yield curve used by the Bank of England for policy formation. Although they ‘easily’ explain the phenomenon of successive yield curves pivoting about a ‘pegged’ maturity and its market impact, the econometric questions involved are subtle. The last chapter, 23 by Helliwell, studies the effects of domestic fiscal policies on the relative economic performance of the US and Japan before the latter’s economic bubble burst in 1990. He concludes that their asymmetric fiscal policies had substantial current account effects on bilateral trade, but that the concomitant changes in nominal exchange rates transmitted little of these effects.